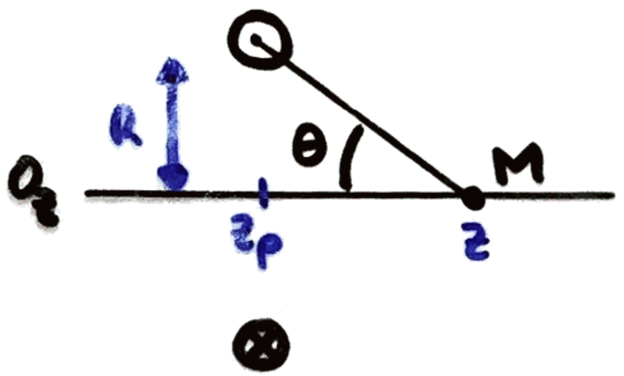
CALCUL DU CHAMP $\vec{B}$

* Les plans passant par Oz sont des plans d'antisymétrie pour $\vec{J}$: $\vec{B} = B_z \vec{e_z}$ (SUR L'AXE)
* invariance par rotation : $B(r,\theta,z) = B(r,z)$
* champ d'une spire : $dB_z(M) = \frac{\mu n I}{2R} \sin^3(\theta)\, d\ell$ I



$$z - z_p = \frac{R}{\tan\theta} \Rightarrow d\ell = dz_p = \frac{R}{\sin^2\theta} d\theta \quad \text{II}$$

$$\text{I et II} \Rightarrow dB_z(M) = \frac{\mu n I}{2} \sin\theta\, d\theta$$

$$\Rightarrow \boxed{B_z(M) = \frac{\mu n I}{2}(\cos\theta_2 - \cos\theta_1)}$$

RELATION CHAMP AXIAL / RADIAL

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\frac{1}{r}\frac{\partial r B_r}{\partial r} + \frac{1}{r}\cancel{\frac{\partial B_\theta}{\partial \theta}} + \frac{\partial B_z}{\partial z} = 0$$

$$\partial(r B_r) = \frac{dB_z}{dz} \cdot (-r\,dr)$$

$$\boxed{B_r = -\frac{r}{2}\frac{dB_z}{dz}}$$

$$\boxed{B_z = \frac{\mu n I}{2}(\cos\theta_2 - \cos\theta_1)}$$

$$\theta_{12} = \frac{\pi}{2} - \mathrm{Atan}\left(\frac{z + L/2}{R}\right)$$

$$\frac{z}{R} + \frac{L}{2R} = \tan\left(\frac{\pi}{2} - \theta_{12}\right)$$

$$dz = \frac{-R}{\sin^2\theta_{12}}\,d\theta_{12}$$

$$\frac{dB_z}{dz} = \frac{\mu n I}{2}\left(\frac{\cos\theta_2}{d\theta}\frac{d\theta}{dz} - \frac{\cos\theta_1}{d\theta}\frac{d\theta}{dz}\right)$$

$$\frac{dB_z}{dz} = \frac{\mu n I}{2R}(\sin^3\theta_1 - \sin^3\theta_2)$$

$$\boxed{B_r = \frac{\mu n I r}{2R}(\sin^3\theta_2 - \sin^3\theta_1)}$$

CALCUL DU COURANT INDUIT

* On cherche $\vec{A}$ tq: $\vec{\nabla} \wedge \vec{A} = \vec{B}$

$\hookrightarrow$ $\boxed{\vec{A} = (\vec{B} \cdot \vec{e_z}) \frac{r}{2} \vec{e_\theta}}$ *

* $\vec{j} = \gamma \vec{E} = -\gamma \frac{\partial \vec{A}}{\partial t}$

$= -\frac{\gamma r}{2} \frac{\partial B_z}{\partial t} \vec{e_\theta}$

$$\boxed{\vec{j} = -\gamma \frac{\mu n I' r}{4} (\cos \alpha_1 - \cos \alpha_2) \vec{e_\theta}}$$

$\vec{\nabla} \wedge \vec{A} = -\frac{\partial A_\theta}{\partial z} \vec{e_r} + \frac{1}{r} \frac{\partial r A_\theta}{\partial r} \vec{e_z}$

$= -\frac{r}{2} \frac{dB_z}{dz} \vec{e_r} + B_z \vec{e_z}$

$= B_r \vec{e_r} + B_z \vec{e_z}$

$\boxed{\vec{\nabla} \wedge \vec{A} = \vec{B}}$

EQUATION DU MOUVEMENT

$$\boxed{d\vec{F} = \vec{j}\, d\tau \wedge \vec{B}}$$

$$d\vec{F} = j_\theta\, dr\, dx (2\pi r) \cdot B_r \cdot (\vec{e_\theta} \wedge \vec{e_r})$$

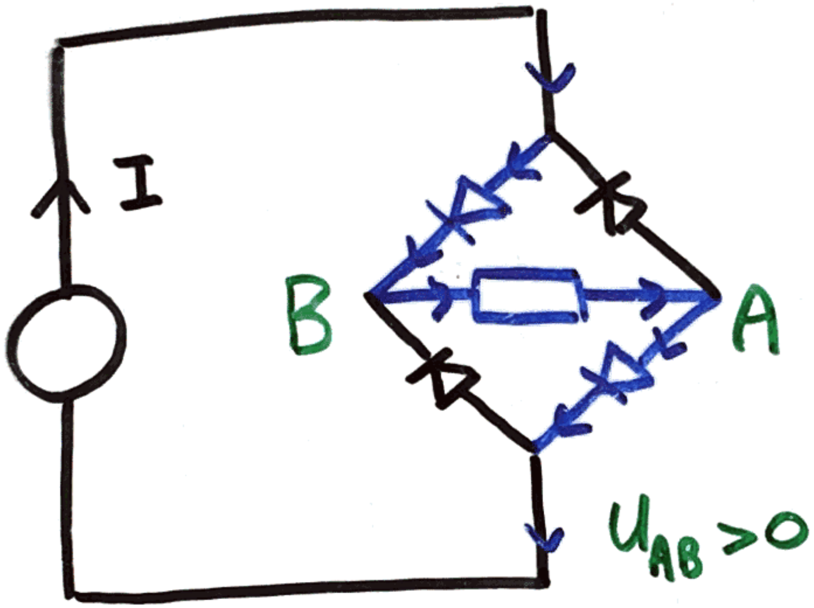
$$= \frac{\mu n I \pi r^2}{2R} (\sin^3\alpha_1 - \sin^3\alpha_2) \cdot j\, dr\, dx\, \vec{e_z}$$

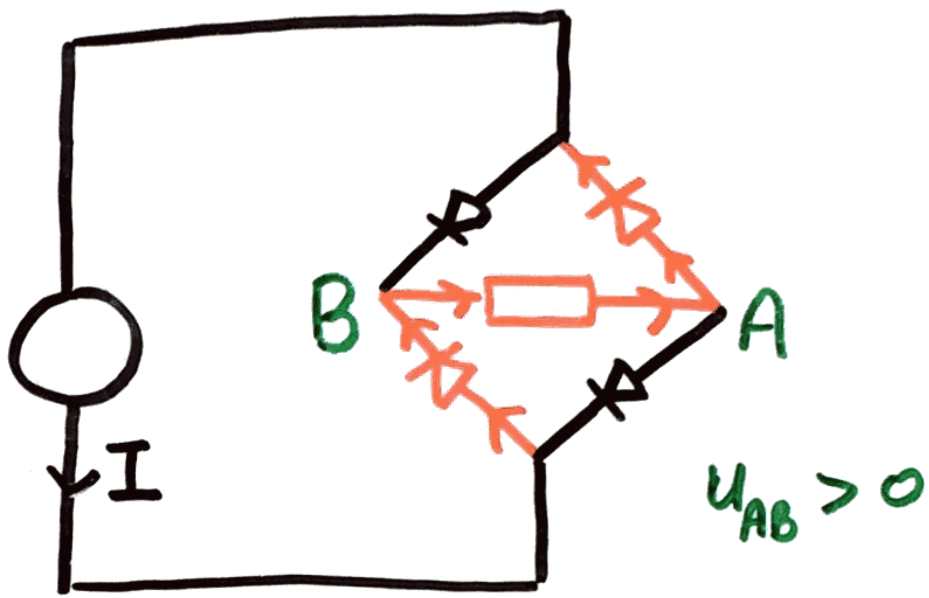
$$d\ddot{z} = \frac{(\mu n)^2 \pi r^3}{8Rm} \cdot I(t) \cdot I'(t) \cdot (\cos\alpha_1 - \cos\alpha_2)(\sin^3\alpha_1 - \sin^3\alpha_2)$$

$$\boxed{\ddot{z} = K \cdot I(t) \cdot I'(t) \cdot \underbrace{\int_{z-L/2}^{z+L/2} (\cos\alpha_1 - \cos\alpha_2)(\sin^3\alpha_1 - \sin^3\alpha_2)\, dx}_{g(z)}}$$

$$\alpha_1 = \frac{\pi}{2} - \mathrm{Atan}\left(\frac{z+L/2}{R}\right)$$

$$\alpha_2 = \frac{\pi}{2} - \mathrm{Atan}\left(\frac{z-L/2}{R}\right)$$


I
B
A
$U_{AB} > 0$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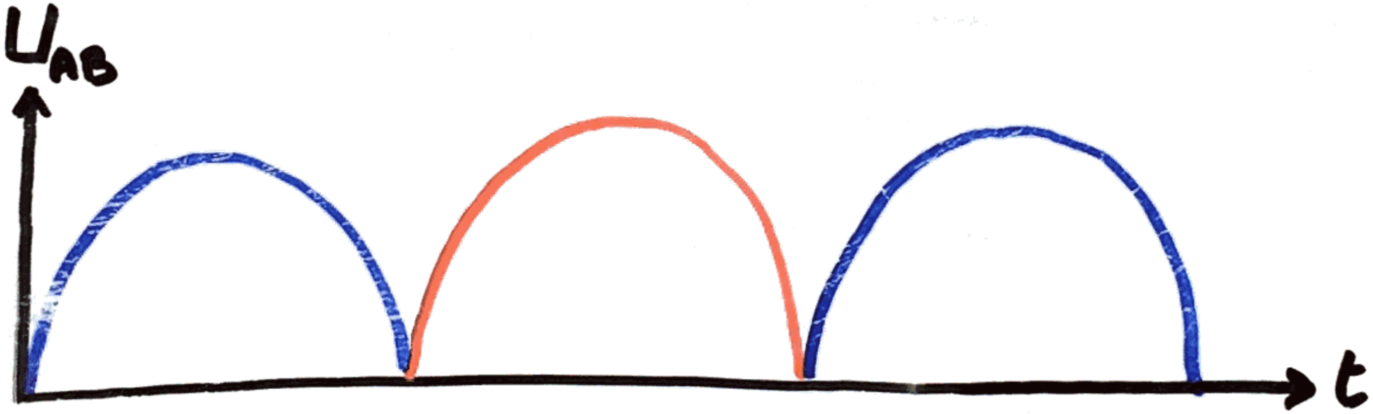
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