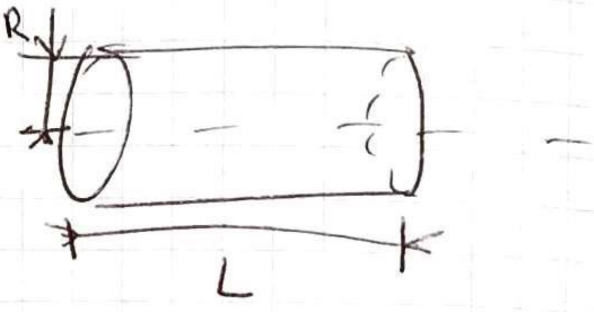
# NEW THEORY: (April 2015).

$$B_z(z) = \mu \frac{nI}{2} (\cos \alpha_1 - \cos \alpha_2)$$

$$\alpha_1 = \frac{\pi}{2} - \mathrm{Atan}\left(\frac{z + L/2}{R}\right)$$

$$\alpha_2 = \frac{\pi}{2} - \mathrm{Atan}\left(\frac{z - L/2}{R}\right)$$




$$\frac{z}{R} + \frac{L}{2R} = \tan\left(\frac{\pi}{2} - \alpha_1\right)$$

$$\frac{dz}{d\alpha_{12}} = \frac{-R}{\cos^2\left(\frac{\pi}{2} - \alpha_{12}\right)}$$

$$dz = -\frac{R\, d\alpha_{12}}{\sin^2 \alpha_{12}}$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow B_r = -r \frac{dB_z}{dz}.}$$

$$B_r = -\frac{r \mu n I}{2}\left(\sin \alpha_1 \frac{d\alpha_1}{dz} - \sin \alpha_2 \frac{d\alpha_2}{dz}\right)$$

$$B_r = \frac{r \mu n I}{2R}\left(\sin^3 \alpha_1 - \sin^3 \alpha_2\right).$$

$\underline{dm}$ : moment magnétique d'un anneau de projectile $(r)$.

$dm = \pi r^2 (\vec{j_0} \cdot \vec{e_\theta})\, dr\, dx$.

$\underline{\vec{j_\theta}}$ : courant volumique en $(r, z, t)$.

comme $\vec{A} = \vec{B}\, \vec{e_z}\, \frac{r}{2}\, \vec{e_\theta}$ (potentiel vecteur).

vérifie $\vec{\nabla} \wedge \vec{A} = \vec{B}$ ici

et $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$ (pas de charges statiques).

et $\vec{j} = \gamma \vec{E} = -\gamma \frac{\partial \vec{A}}{\partial t} = -\gamma \frac{r}{2} \frac{\partial B_z}{\partial t} \vec{e_\theta}$

$$\vec{j} = -\frac{\gamma r}{2} \frac{\partial B_z}{\partial t} \vec{e_\theta}$$

# 1st Method: Energy ($\vec{F} = -\vec{\nabla} E_p$).

$$dF_z = \frac{\partial}{\partial z}(\vec{dm} \cdot \vec{B_z}) = \frac{\partial}{\partial z}(dm \cdot B_z).$$

$$= \frac{\partial}{\partial z}\Big[\underbrace{\pi r^2 (\vec{j_\theta} \cdot \vec{e_\theta})\, dr\, dx}_{dm}\, B_z\Big] \quad \boxed{z - L/2 \leq x \leq z + L/2}$$

WHEN INTEGRATING.

$$= \frac{\partial}{\partial z}\Big[\pi r^2 \big(\underbrace{-\gamma \frac{r}{2} \frac{\partial B_z}{\partial t}}_{j_\theta}\big) dr\, dx\, B_z\Big]$$

$$= -\frac{\pi r^3 \gamma}{2} \frac{\partial}{\partial z}\Big[\frac{\partial B_z}{\partial t} B_z\Big]\, dr\, dx$$

$$= -\frac{\pi r^3 \gamma}{2}\left(\frac{\mu n}{2}\right)^2 \cdot I \cdot I' \cdot \frac{d}{dz}\Big[(\cos\alpha_1 - \cos\alpha_2)^2\Big]\, dr\, dx$$

$$= +\frac{\pi r^3 \gamma}{8R}(\mu n)^2 \cdot I \cdot I' \cdot (\cos\alpha_1 - \cos\alpha_2)(\sin^3\alpha_1 - \sin^3\alpha_2)\, dr\, dx$$

$$\Rightarrow d\ddot{z} = +\frac{\pi r^3 \gamma (\mu n)^2}{8Rm} \cdot I \cdot I' \cdot (\cos\alpha_1 - \cos\alpha_2)(\sin^3\alpha_1 - \sin^3\alpha_2)\, dr\, dx.$$

$$\Rightarrow \boxed{\ddot{z} = K \cdot f(t) \cdot \int_{z-L/2}^{z+L/2} g(x)\, dx}$$

$f(t) = I(t) \cdot I'(t)$.

$g(z) = (\cos\alpha_1 - \cos\alpha_2)(\sin^3\alpha_2 - \sin^3\alpha_2)$

## 2nd Method: LAPLACE FORCE ($\vec{F} = \vec{j}\, d\tau \wedge \vec{B}$)

$$\vec{dF} = (-j\, dr\, dx\, \vec{e_\theta}\, 2\pi r) \wedge (B_r\, \vec{e_r})$$

$$= \frac{\mu n I \pi r^2}{2R} (\sin^3\alpha_1 - \sin^3\alpha_2)\, \overbrace{j}^{\frac{\gamma r}{2} \frac{\partial B_z}{\partial t}}\, dr\, dx\, \vec{e_z}$$

Sur $\vec{e_z}$:

$$d\ddot{z} = \frac{(\mu n)^2 \pi r^3}{8Rm} (\cos\alpha_1 - \cos\alpha_2)(\sin^3\alpha_1 - \sin^3\alpha_2)\, dr\, dx\ I \cdot I'.$$

$$\boxed{\ddot{z} = k \cdot f(t) \cdot \int_{z-l/2}^{z+l/2} g(x)\, dx}$$